

MOND and the seven dwarfs

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ABSTRACT

Gerhard (1994) had recently analyzed the data on seven dwarf spheroidals, and concluded that these disagree with the predictions of MOND. We contend that this conclusion is anything but correct. With new data for three of the dwarfs the observations are in compelling agreement with the predictions of MOND. Gerhard found MOND M/L values that fall around a few solar units, as expected if MOND is a valid alternative to dark matter—while the Newtonian M/L values are in the range of about 10 to 100 solar units. Gerhard’s sole cause for complaint was that some of his MOND M/L values were still outside the range of “reasonable” stellar values, say between 1 and 3 (two were too high—about 10—and one too low). This, we say, was *easily* attributable to uncertainties in the data, such as in the velocity dispersions, luminosities, core radii, etc., and in the assumptions that underlie the analysis, such as isotropic velocity distribution, light-distribution fits, no tidal effects, etc.. There are now new, much-improved determinations of the velocity dispersions for three of the dwarfs, in particular for the two dwarfs for which Gerhard found high MOND M/L values: The preliminary, high value for Leo II has been superseded by a much smaller value (by a factor of two) that gives a MOND M/L value of about 1.5 (instead of about 9). There is also a new determination of the velocity dispersion of Ursa Minor which brings down the MOND M/L value considerably. The one small MOND M/L value that Gerhard found is also shown to be a red herring. In fact, within just the quoted errors on the velocity dispersions and the luminosities, the MOND M/L values for all seven dwarfs are perfectly consistent with stellar values, with no need for dark matter.

Subject headings: Dwarf spheroidals, dark matter, modified dynamics

1. Introduction

Low-surface-density galaxies provide crucial test cases for the modified dynamics—MOND (Milgrom 1983a,b). Low surface density is tantamount to low accelerations, and it is the basic tenet of MOND that low-acceleration systems must evince large mass discrepancies (or in the

common way of thinking, large amounts of dark matter). Relevant systems include very diverse types, such as dwarf spirals, dwarf irregulars, low-surface-brightness (LSB) galaxies of normal or large size, and the dwarf-spheroidal (DS) neighbors of the Milky way.

Long before relevant data was available for any of these classes, Milgrom (1983b) predicted that such galaxies will show large mass discrepancies; and that, unlike higher-surface-density galaxies, the discrepancy will appear starting from the centre of the galaxy, not only beyond some transition radius, as the accelerations in such systems are small already at the centre. These predictions have been amply born out by recent observations of dwarf spirals [Carignan, Sancisi, and van Albada 1988, Milgrom and Braun (1988), Carignan and Beaulieu (1989), Jobin and Carignan (1990), Lake, Schommer, and van Gorkom (1990), Puche, Carignan, and Bosma (1990), Begeman, Broeils, and Sanders (1991)], of dwarf irregulars [Lo, Sargent, and Young (1993), Milgrom (1994)], and of LSB galaxies. For a sample of six of these last (the ilk of Malin 1) the results of van der Hulst et al. (1993) imply M/L values roughly between 10 and 75 solar units, at the last measured points on the rotation curves (with large uncertainties); MOND analysis gives values consistent with values of a few solar units (Milgrom, unpublished).

About dwarf spheroidals—which are the subject of this paper—we find in Milgrom (1983b): “...we predict that when velocity-dispersion data is available for the (spheroidal) dwarfs, a large mass discrepancy will result when the conventional dynamics is used to determine the masses. The dynamically determined mass is predicted to be larger by a factor of order 10 or more than that which is accounted for by stars.” This prediction was made solely on the basis of the low surface brightness of the dwarfs.

We know now that, indeed, the Newtonian M/L values deduced for DS are, typically, between 10 and 100 solar units. [For recent compilations and further references to earlier work see Mateo (1994), and Vogt et al. (1995).] These are still very uncertain due to both observational and procedural uncertainties [see e.g. Lake (1990), Pryor and Kormendy (1990), Pryor (1994)]. Indeed, the very assumption of virial equilibrium, which underlies the mass determination, has been questioned.

If the dynamical mass in the dwarfs is dominated by stars, then, if we determine the masses of the dwarfs using MOND we should get M/L values that are typical of their stellar populations.

Applying MOND analysis to seven DS (Draco, Carina, Ursa Minor, Sextans, Sculptor, Fornax, and Leo II) Gerhard (1994) claims that the predictions of MOND for these dwarfs are in conflict with the observations. His analysis extends and updates an analysis for five of the dwarfs by Gerhard and Spergel (1992), who reach qualitatively similar, unfavorable conclusions.

Quoth Gerhard (1994): “Two results are immediately apparent: Objects like Fornax require very low M/L in MOND, while others like UMi still require significant dark matter. Between the dwarf spheroidals, M/L must vary by a factor of at least 20 even in MOND.”

In fact, “objects like Fornax” stands for Fornax alone, for which Gerhard found central M/L

values between 0.2 and 0.3; and “objects like UMi” stands for UMi ($M/L \sim 10 - 13$) and Leo II ($M/L \sim 9$). For Carina, Sextans, and Sculptor Gerhard found, by the appropriate MOND estimator, M/L values between 1 and 3, and for Draco he found $M/L \sim 6$.

Even with these results, Gerhard’s adverse conclusion was unwarranted, as he has been well aware of the large uncertainties still besetting the analysis. We discuss these in more detail below, and show that within the quoted uncertainties available to Gerhard (1994) his MOND M/L values were consistent with stellar values (he does not discuss the error range).

In vindication of the above statement there exist now new, high-quality determinations of the velocity dispersions that improve dramatically the agreement with MOND: (i) The determination of the dispersion for Leo II by Vogt et al. (1995) that supersedes the preliminary value [Mateo (1994)] used by Gerhard (1994) gives a MOND M/L value of about 1.5, instead of the 9 that Gerhard obtained. (ii) A new determination of the dispersion in UMi [Hargreaves et al. (1994b)] that is much smaller than that used by Gerhard, and which gives UMi a stellar MOND M/L value. (iii) An updated value of σ for Draco would have given Gerhard an M/L value of 4 instead of 6.

All in all, the agreement with MOND is now as good as could be expected in light of the remaining uncertainties.

In §2 we briefly describe the various MOND M/L estimators, and discuss the uncertainties involved in their determination. In §3 we consider individual dwarfs with their idiosyncrasies.

2. MOND M/L estimators and their uncertainties

Gerhard and Spergel (1992), and Gerhard (1994) consider four types of MOND M/L estimators; two of them concern central values, and two global values—the difference being analogous to that in the Newtonian case. Within each pair, one estimator is based on the assumption that the dwarf is isolated and unaffected by the external field of the Milky Way; we refer to this as the global value. The other estimator is based on the oppositely extreme assumption that the dwarf is dominated by the external field: the acceleration with which the system falls in the external field is large compared with internal accelerations. In the latter case, the internal dynamics in MOND is very nearly Newtonian, with an effective gravitational constant that is determined by the external acceleration [Milgrom (1986)]—we call this limit, after Gerhard and Spergel (1992), the quasi-Newtonian case. All accelerations relevant to the dwarf dynamics are much smaller than the acceleration constant a_o . In this case the effective gravitational constant in the quasi-Newtonian limit is simply $G a_o / a_{ex}$, where a_{ex} is the acceleration of the system in the external field. Thus, in this case, the MOND M/L value is obtained by multiplying the Newtonian value by a_{ex} / a_o . We can write $a_{ex} = V^2 / R$, where R is the galactocentric distance of the dwarf,

and V is the rotational velocity at the position of the dwarf. As did Gerhard and Spergel (1992), and Gerhard (1994), we take $V = V_\infty = 220 \text{ km s}^{-1}$ for all the dwarfs.

A measure of the degree of isolation in the above sense is provided by the parameter

$$\eta \equiv \frac{3\sigma^2/2r_c}{V_\infty^2/R} \approx \frac{a_{in}}{a_{ex}}, \quad (1)$$

where r_c is the core radius, and σ is the (mean) line-of-sight velocity dispersion.

Some of the dwarfs are borderline, with $\eta \sim 1$. In this case the two estimators (quasi-Newtonian, and isolated) should give similar M/L values. Neither limit is valid in this case, but the correction to M/L due to this fact is not large (it works to increase M/L somewhat, by at most about 40 percent).

For the isolated case Gerhard and Spergel (1992) use an estimator of the central, MOND M/L value of the form $M/L \propto \sigma_0^4/a_o I(0)r_c^2$, where σ_0 is the central velocity dispersion, and $I(0)$ is the central surface brightness. For the global estimator one uses the MOND mass-velocity relation

$$M = \frac{9}{4} \frac{\sigma_3^4}{Ga_o}, \quad (2)$$

where σ_3 is the *three-dimensional* rms velocity dispersion for the whole system. Milgrom (1994) derived this relation for an arbitrary, low-acceleration ($a_{in} \ll a_o$), stationary, isolated system in the formulation of MOND given by Bekenstein and Milgrom (1984); it had been derived earlier for spherical systems by Gerhard and Spergel (1992) [and earlier yet for isothermal spheres in Milgrom (1984)].

If the system is globally isotropic, i.e. has the same line-of-sight rms dispersion, σ , in all directions, we have $\sigma_3 = 3^{1/2}\sigma$ and can write

$$M = \frac{81}{4} \frac{\sigma^4}{Ga_o}; \quad (3)$$

this is the relation used all along for the global isolated case.

Regarding uncertainties, MOND estimators are, naturally, free of the uncertainly related to the unknown distribution of the dark matter relative to the visible one. In fact, in the quasi-Newtonian case the assumption that “mass follows light” is appropriate, as the MOND effect here is predominantly to increase the value of the gravitational constant. The quasi-Newtonian estimators are, otherwise, subject to the same uncertainties that beset the Newtonian analysis, and which have been much emphasized in the literature [e.g. Lake (1990), Pryor and Kormendy (1990), Caldwell et al. (1992), Pryor (1994), Vogt et al. (1995)].

Among the observational parameters of the dwarfs, the photometric (structural) parameters—central surface brightness, core radius, tidal radius, total luminosity—are notoriously difficult to determine. Caldwell et al. (1992)—whence come much of the photometric parameters used by

Gerhard (1994)–discuss these problems in detail. The velocity dispersions had also been rather uncertain, but there seems to be rapid and significant progress in this regard with many more stellar velocities measured, and increased control of contribution from binaries. The isolated MOND estimators are more sensitive to uncertainties in the dispersions as they are proportional to the fourth power of the latter. The central MOND estimators (isolated or quasi-Newtonian) are very sensitive to assumptions on the unknown distribution of orbit orientations in the system in analogy to the Newtonian case [see e.g. Lake (1990), Pryor and Kormendy (1990)]. Most of the estimates mentioned below assume an isotropic distribution. The global, isolated MOND estimator does not require knowledge of the orbit distribution, but makes use of the rms dispersion for the whole system, when many times only central values are available. In only two cases (Fornax and Leo II) are there, at the moment, some assurances that the quoted value can be used as the rms dispersion. The global luminosity value is, on the other hand, less certain than the central luminosity (say within the half brightness radius), which enter the determinations of central values.

The possible presence of tidal-disruption effects on the velocity dispersions has also been raised: At least some of the dwarfs may be undergoing tidal disruption to some degree, a reckoning without which fact would lead to an overestimate of the mass. Gerhard (1994), and Vogt et al. (1995) summarize the pros and cons of this actually being an important effect, and give references to earlier discussions. The most cogent counter argument is that no systematic gradients of velocities are found as would be expected if tidal disruption is responsible for much of the velocity dispersion. Mateo et al. (1993) define a parameter X that measures the potential susceptibility of the dwarf to tidal effects (ratio of the central density in the dwarf to the average galactic density within the galactocentric distance of the dwarf). In the analysis we ignore such effects.

3. MOND M/L values of individual dwarfs

Here we estimate the MOND M/L values of the individual dwarfs. We use all along $a_o = 2 \cdot 10^{-8} \text{ cm s}^{-2}$, as did Gerhard and Spergel (1992) and Gerhard (1994), to facilitate comparison (all derived M/L values scale like a_o^{-1}). Unless stated otherwise we take all dwarf parameters as used by Gerhard (1994) [who, in turn, takes them from Mateo (1994)].

Sculptor

This dwarf has $\eta \approx 0.5$, which points to domination by the external field. For this situation Gerhard found a central, MOND $M/L = 1.2$, based on the Newtonian value $(M/L)_N = 12$ [Mateo (1994)]. The range resulting from errors in σ alone– $\sigma = 6.8 \pm 0.8 \text{ km s}^{-1}$ –is $0.9 \leq M/L \leq 1.5$. The central surface brightness comes originally from Caldwell et al. (1992), who give an error estimate of $\pm 0.3 \text{ mag/arcsec}^2$, increasing the range to $0.7 \leq M/L \leq 2.0$.

Sextans

With the quoted dispersion, $\sigma = 6.2 \pm 0.8$ this has an $\eta \approx 0.3$, and is thus most probably dominated by the external field. It is also among those with the poorest isolation value by the X criterion. For this case Gerhard (1994) found a central, MOND M/L of 1.5 [based on a Newtonian value of 18 from Mateo (1994), who, in turn, quotes Mateo et al. (1993)]. The range corresponding to the error in σ is $1.1 \leq M/L \leq 1.9$. The value of the central surface brightness used by Gerhard originates with Caldwell et al. (1992) who had estimated the error to be ± 0.5 mag/arcsec², further broadening the range to $0.7 \leq M/L \leq 3.0$.

Note that there are significant differences between the structural parameter values used by different authors. We, with Gerhard (1994), use those given in Mateo (1994), Mateo et al. (1993); but compare Suntzeff et al. (1993), and Hargreaves et al. (1994a).

Carina

Here $\eta \approx 0.6$. For the external-field-dominated case, which seems more appropriate, Gerhard found a MOND, central value $M/L = 3.6$ (1.8 for the isolated case). This corresponds to the central value of $\sigma = 6.8 \pm 1.6$ [Mateo et al. (1993), Mateo (1994)]. The range corresponding to the error on σ alone is $2.1 \leq M/L \leq 5.5$. The quoted relative error on the central luminosity surface density is ± 0.3 mag/arcsec² [Mateo et al. (1993)], further broadening this range to $1.6 \leq M/L \leq 7.2$. The Newtonian, V-band, central M/L value from which these are derived [Mateo et al. (1993)] is $(M/L)_N = 39 \pm 23$.

Draco

With the value of the dispersion used by Gerhard (1994), $\sigma = 10.2 \pm 1.8$ km s⁻¹, this dwarf has $\eta \approx 1.3$ and it is plausible to treat it as isolated, for which case Gerhard gets a central MOND value of $M/L = 6.1$ (5.8 for the quasi-Newtonian case). The range that corresponds to the quoted measurement errors on σ alone is $2.8 \leq M/L \leq 11.7$. [Gerhard and Spergel (1992) had obtained for this dwarf MOND M/L values of 17 and 10 instead of the 6.1 and 5.8 that Gerhard found—highlighting the state of flux the analysis had been in.]

Recently, Pryor, Olszewski, and Armandroff (1995) have obtained a somewhat smaller value for the dispersion, with a much smaller error, based on measurements of many more stellar velocities: $\sigma = 9.2 \pm 0.8$, which gives an isolated MOND $M/L = 4$, with a range $2.8 \leq M/L \leq 5.6$, corresponding to the error on σ alone (I could not find an error quoted for the central surface brightness).

Leo II

Gerhard (1994) used the preliminary value of $\sigma = 13.8$ km s⁻¹ given in Mateo (1994), to obtain an isolated, central, MOND M/L of 9.3. However, the quoted error on σ was ± 5.5 km s⁻¹ allowing an M/L value as small as 1.2.

At any rate, there is now a new determination of the dispersion of Leo II that definitely supersedes the above mentioned. Vogt et al. (1995) find $\sigma = 6.7 \pm 1.1$ km s⁻¹; they give this as

the rms for Leo II, not the central value. Dividing the data between three radial bins Vogt et al. (1995) find no significant change of σ with radius. We thus use it to derive the more sound, global M/L value using relation (3).

This dwarf has $\eta = 1.4$ (with the updated σ), and is the most isolated in the MOND sense (with respect to the external acceleration). It is also, by far, the least likely to be affected by tidal effects. We treat it as isolated, remembering that the effect of the external field, which may be present, would be to increase somewhat the MOND M/L value.

The MOND mass for Leo II for the above value of σ is $M = (1.5^{+1.25}_{-0.77})10^6 M_{\odot}$. With the (V-band) luminosity given in Vogt et al. (1995), $L = (9.9 \pm 0.2)10^5 L_{\odot}$, we get a global $M/L = 1.5$ with an error range of between 0.7 and 3.3. The global, Newtonian M/L found by Vogt et al. (1995) is $(M/L)_N = 11.1 \pm 3.8$.

Ursa Minor

For this dwarf Gerhard (1994) uses the value $\sigma = 12.0 \pm 2.4 \text{ kms}^{-1}$ given in Mateo (1994). However, in a more recent paper, Hargreaves et al. (1994b) find a much lower value of $\sigma = 7.5^{+1.0}_{-0.9} \text{ kms}^{-1}$, based on measurements of many more velocities. This smaller value is consistent with the recent result of Pryor, Olszewski, and Armandroff (1995) who find $\sigma = 8.9 \pm 0.8 \text{ kms}^{-1}$. With the high value of σ we get $\eta \approx 1$, which makes UMi a borderline case. Gerhard found, central, MOND M/L values of 10 for the isolated case and 13 for the quasi-Newtonian case. The ranges corresponding to the quoted errors on σ alone are $4.1 \leq M/L \leq 20.7$, and $8.3 \leq M/L \leq 18.7$, respectively. The value of the central surface brightness used by Gerhard originates with Caldwell et al. (1992) who had estimated the error to be $\pm 0.5 \text{ mag/arcsec}^2$, further broadening the error range on M/L . Gerhard himself, in his discussion of Newtonian M/L values for the dwarfs says that the range of values for UMi is about 20–100; yet, for the derivation of his MOND value he used a Newtonian value of 110, at the uppermost end of this range.

With the smaller value of σ we obtain $\eta \approx 0.3$, and it is reasonable to treat UMi as dominated by the external field. In this case we get, updating only σ , and leaving the other parameter values as used by Gerhard, $M/L = 5.1$, instead of 13. The range corresponding to the quoted errors on σ is $3.9 \leq M/L \leq 6.5$. The central, Newtonian value Hargreaves et al. (1994b) find is $(M/L)_N = 59^{+41}_{-25}$. The errors on the central surface brightness, core radius, and total luminosity broadens the above given ranges (to say nothing of the effects of wrong assumptions such as isotropic orbits). For example, the error Hargreaves et al. (1994b) quote for the surface luminosity density is about $\pm 50\%$, which reduces the lower limit on M/L to 2.6. Furthermore, Hargreaves et al. (1994b) find some contribution from rigid rotation to the velocity dispersion—confirmed by Pryor, Olszewski, and Armandroff (1995)—which reduces the value of the pure dispersion to $\sigma = 6.7^{+0.9}_{-0.8} \text{ kms}^{-1}$ further reducing the above estimates of the MOND M/L values.

Fornax

Here $\eta \approx 0.8$, and so Fornax is a borderline case as regards MOND isolation. It is deemed

relatively immune to tidal effects by the X criterion. Fornax is also unique among the seven in that line of sight dispersions have been measured to relatively large radii. Preliminary analysis of Mateo and Olszewski (private communication) shows that σ remains constant down to the tidal radius. Thus, we can safely use the MOND global mass-velocity relation eq.(3) that is less uncertain than the relation employing central quantities. Assuming an isolated Fornax we get from eq.(3), and the quoted $\sigma = 11 \pm 2 \text{ km s}^{-1}$, a MOND mass of $M = 1.1 \cdot 10^7 M_\odot$, with a range corresponding to the quoted error on σ of $(0.5 - 2.2) \cdot 10^7 M_\odot$. We find in the literature rather different values quoted for the (V-band) luminosity of Fornax. Gerhard (1994) uses $L_V = 2.5 \cdot 10^7 M_\odot$ which originates with Caldwell et al. (1992) who estimate the error to be $\pm 5 \text{ mag/arcsec}^2$; Gerhard and Spergel (1992) use a value three times smaller of $L_V = 8 \cdot 10^6 M_\odot$ quoted from Pryor (1992). With the former value of L the range of MOND M/L values is 0.2–0.9 with only the error on σ , and 0.1 – 1.4 with the error on L included. With the latter value of L the range is 0.6–2.7.

Gerhard (1994) does not give global values, but Gerhard and Spergel (1992) do, and they find, like us, values of order 1 (with a somewhat smaller dispersion of 10 km s^{-1}): 0.95 for the isolated case, and 0.74 for the purely quasi-Newtonian case.

Gerhard (1994) found small MOND M/L values (0.2-0.3), based on a Newtonian value of about 5. Some idea of the uncertainties can be gotten from the range of acceptable Newtonian values Mateo et al. (1991) cite: $5.3 \leq (M/L)_N \leq 26$, stating that this large range is dominated by uncertainties in the structural parameters of Fornax. No improvement of the situation has since been made in this regard. The low MOND M/L values Gerhard found are based on a Newtonian value at the very low end of this range.

Mateo et al. (1991) argue that Fornax must have a stellar M/L value “about half that observed in globular clusters or in the range 0.5–1.5”, because it is characterized by a relatively young population, which is expected to have rather smaller M/L values. The MOND M/L values we find and even the lower values Gerhard found are, when considering the uncertainties, definitely consistent with such stellar values.

In summary of this section, we find that with the updated values of the velocity dispersions, and within just the quoted measurement errors on the velocity dispersions and luminosities, the observations of all seven dwarfs are perfectly consistent with MOND stellar M/L in the range of 1 to 3 solar units. The kinematics of the dwarfs can thus be understood with MOND dynamics and with no need for dark matter.

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